# On Radio D-distance Number of some basic Graphs

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***Abstract*— In this paper we find the radio D-distance number of some standard graphs. If u, v are vertices of a connected graph *G*, the *D*-length of a connected u-v path *s* is defined as $l^D(s) = l(s) + \deg(v) + \deg(u) + \sum \deg(w)$, where the sum runs over all intermediate vertices *w* of *s* and *l*(*s*) is the length of the path. The *D*-distance $d^D(u, v)$ between two vertices u, v of a connected graph G is defined a $d^D(u, v) = \min\{l^D(s)\}$, where the minimum is taken over all u-v paths *s* in *G*. In other words, $d^D(u, v) = \min\{l(s) + \deg(v) + \deg(u) + \sum \deg(w)\}$, where the sum runs over all intermediate vertices *w* in *s* and minimum is taken over all u-v paths *s* in G. Radio D-distance coloring is a function $f : V(G) \rightarrow \mathbb{N}$ such that $d^D(u, v) + |f(u) - f(v)| \geq diam^D(G) + 1$, where $diam^D(G)$ is the D-distance diameter of G. A D-distance radio coloring number of G is the maximum color assigned to any vertex of G. It is denoted by $rn^D(G)$.**

***Keywords*— D-distance, Radio D-distance coloring, Radio D-distance number***

## I. Introduction

By a graph G = (V (G), E (G)) we mean a finite undirected graph without loops or multiple edges. The order and size of G are denoted by p and q respectively.

If u, v are vertices of a connected graph *G*, the *D*-length of a connected u-v path *s* is defined as $l^D(s) = l(s) + \deg(v) + \deg(u) + \sum \deg(w)$, where the sum runs over all intermediate vertices *w* of *s* and *l*(*s*) is the length of the path. The *D*-distance $d^D(u, v)$ between two vertices u, v of a connected graph G is defined a $d^D(u, v) = \min\{l^D(s)\}$, where the minimum is taken over all u-v paths *s* in *G*. In other words, $d^D(u, v) = \min\{l(s) + \deg(v) + \deg(u) + \sum \deg(w)\}$, where the sum runs over all intermediate vertices *w* in *s* and minimum is taken over all u-v paths *s* in G. Radio D-distance coloring is a function $f : V(G) \rightarrow \mathbb{N}$ such that $d^D(u, v) + |f(u) - f(v)| \geq diam^D(G) + 1$, where $diam^D(G)$ is the D-distance diameter of G. A D-distance radio coloring number of G is the maximum color assigned to any vertex of G. It is denoted by $rn^D(G)$. The D-distance was introduced by Reddy Babu et al. [18, 19, 20].

Let G be a connected graph of diameter d and let k an integer such that $1 \leq k \leq d$. A radio k-coloring of G is an assignment *f* of colors (positive integers) to the vertices of G such that $d(u, v) + |f(u) - f(v)| \geq 1 + k$ for every two distinct vertices u, v of G. The radio k-coloring number $rc_k(f)$ of a radio k-coloring *f* of G is the maximum color assigned to a vertex of G. The radio k-chromatic number $rc_k(G)$ is $\min\{rc_k(f)\}$ over all radio k-colorings *f* of G. A radio k-coloring *f* of G is a minimal radio k-coloring if $rc_k(f) = rc_k(G)$. A set S of positive integers is a radio k-coloring set if the elements of S are used in a radio k-coloring of some graph G and S is a minimum radio k-coloring set if S is a radio k-coloring set of a minimum radio k-coloring of some graph G. The radio 1-chromatic number $rc_1(G)$ is then the chromatic number $\chi(G)$. When k = Diam(G), the resulting radio k-coloring is called radio coloring of G. The radio number of G is defined as the minimum span of a radio coloring of G and is denoted as rn(G).

Radio labeling can be regarded as an extension of distance-two labeling which is motivated by the channel assignment problem introduced by W. K. Hale [6]. G. Chartrand et al.[2] introduced the concept of radio labeling of graph. Also G. Chartrand et al.[3] gave the upper bound for the radio number of path. The exact value for the radio number of path and cycle was given by Liu and Zhu [10]. However G. Chartrand et al.[2] obtained different values for them. They found the lower and upper bound for the radio number of cycle. Liu [9] gave the lower bound for the radio number of Tree. The exact value for the radio number of Hypercube was given by R. Khennoufa and O. Togni [8]. M. M. Rivera et al. [21] gave the radio number of $C_n \times C_n$, the Cartesian product of $C_n$. In [4] C. Fernandez et al. found the radio number for complete graph, Star graph, Complete Bipartite graph, Wheel graph and Gear graph. M. T. Rahim and I. Tomescu [17] investigated the radio number of Helm graph. The radio number for the generalized prism graphs were presented by Paul Martinez et al. in [11]. In this paper, we fined the radio D-distance coloring of some standard graphs.

Definition [12]: The radio D-distance coloring is a function $f : V(G) \rightarrow \mathbb{N}$ such that $d^D(u, v) + |f(u) - f(v)| \geq diam^D(G) + 1$, where $diam^D(G)$ is the D-distance diameter of G. A radio D-distance coloring number of G is the maximum color assigned to any vertex of G. It is denoted by

$rc^D(G)$. In this paper, we find the radio D-distance number of some graphs.

## II. Main Results

**Theorem 2.1**

The radio D-distance number of complete graph $K_n$, $rn^D(K_n) = n$.

**Proof**

Since $diam^D(G) = d^D(u, v)$ for any u, v $\in V(K_n)$, the radio D-distance condition implies $|f(u) - f(v)| \geq 1$ for all u, v $\in V(K_n)$.

Since f: $V(K_n) \to \mathbb{N}$ is injective, it follows that $rn^D(K_n) \leq$ n. Hence $rn^D(K_n)$ = n, since $|V| \leq rn^D(K_n) \leq$ n

**Theorem 2.2**

The radio D-distance number of star graph $K_{1,n}$, $rn^D(K_{1,n})$ = n + 3, if n $\geq$ 2.

**Proof**

Let $V(K_{1,n}) = \{v_0, v_1, v_2, \ldots, v_n\}$ be vertex set, where $v_0$ is the apex vertex and $E(K_{1,n}) = \{v_0 v_i$ for all i = 1, 2, . . . , n} be edge set. Then $d^D(v_0, v_i)$ = n + 2, $1 \leq i \leq$ n, $d^D(v_i, v_j)$ = n + 4, $1 \leq i, j \leq$ n, So $diam^D(K_{1,n})$ = n + 4.

The radio D-distance condition becomes

$d^D(v_i, v_j) + |f(v_i) - f(v_j)| \geq$ n + 5, for any vi, vj $\in V(K_{1,n})$,Now, $d^D(v_0, v_n) + |f(v_0) - f(v_n)| \geq$ n + 5

Therefore, $rn^D(K_{1,n})$ = n + 3 if n ≥ 2.

**Theorem 2.3**

The radio D-distance number of Book with triangle page graph $K_2 + mK_1$,

$rn^D(K_2 + mK_1) = m^2 - 3m + 5$ if $m \geq 5$

**Proof**

Let $V(K_2 + mK_1) = \{v_1, v_2, u_1, u_2, u_3, \ldots, u_m\}$ be vertex set and $E(K_2 + mK_1) = \{v_1 v_2, v_1 u_i, v_2 u_i$, for i = 1, 2, . . . , n}. Then $d^D(u_i, u_j)$ = m + 7, $1 \leq i,j \leq$ m and $d^D(v_i, u_j)$ = m + 4, $i = 1, 2$ and $1 \leq j \leq$ m and $d^D(v_1, v_2)$ = 2m + 3.

Then $diam^D(K_2 + mK_1)$ = 2m + 3.

Let $f(v_1) < f(v_2) < f(u_1) < f(u_2) < \ldots < f(u_{m-1}) < f(u_m)$

The radio D-distance condition is

$d^D(v_1, v_2) + |f(v_1) - f(v_2)| \geq$ 2m + 4

$d^D(v_2, u_1) + |f(v_2) - f(u_1)| \geq$ 2m + 4.

$d^D(u_1, u_2) + |f(u_1) - f(u_2)| \geq$ 2m + 4

Define $f(u_i) = i(m - 3) + 5$, $1 \leq i \leq$ m.

Hence, $rn^D(K_2 + mK_1) = m^2 - 3m + 5$.

**Note**

$rn^D(K_2 + mK_1)$ = 9 if $2 \leq m \leq 4$.

**Theorem 2.4**

The radio D-distance number of bistar $B_{n,n}$, $rn^D(B_{n,n}) = n^2$ + 3n + 8 , n $\geq$ 2.

**Proof**

Let $V(B_{n,n}) = \{v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n, x_1, x_2\}$ be vertex set, where $x_1$, $x_2$ are the central vertices. $E(B_{n,n}) = \{x_1 x_2, x_1 v_i, x_2 u_i$ : i = 1, 2, . . . ,n} be edge set. Then $d^D(x_1, x_2)$ = 2n + 3, $d^D(x_1, u_i) = d^D(v_i, x_2)$ = 2n + 5, $1 \leq i \leq$ n, $d^D(u_i, u_j) = d^D(v_i, v_j)$ = n + 5, $1 \leq i,j \leq$ n $d^D(v_i, u_i)$ = 2n + 7, $1 \leq i \leq$ n. Then, $diam^D(B_{n,n})$ = 2n + 7.

Let $f(v_1) < f(u_1) < f(v_2) < f(u_2) < \ldots < f(v_m) < f(u_m) < f(x_1) < f(x_2)$.

The radio D – distance condition is

$d^D(v_1, u_1) + |f(v_1) - f(u_1)| \geq 2n + 8$

$d^D(u_1, v_2) + |f(u_1) - f(v_2)| \geq 2n + 8$

and $d^D(v_1, v_2) + |f(v_1) - f(v_2)| \geq 2n + 8$

$d^D(v_2, u_2) + |f(v_2) - f(u_2)| \geq 2n + 8$

and $d^D(u_1, u_2) + |f(u_1) - f(u_2)| \geq 2n + 8$

$f(v_i) = (n + 3)i - (n + 2)$, $f(u_i) = (n + 3)i - (n + 1)$, $1 \leq i \leq$ n.

$d^D(u_n, x_1) + |f(u_n) - f(x_1)| \geq 2n + 8$

and $d^D(v_n, x_1) + |f(v_n) - f(x_1)| \geq 2n + 8$

$d^D(x_1, x_2) + |f(x_1) - f(x_2)| \geq 2n + 8$

and $d^D(u_n, x_2) + |f(u_n) - f(x_2)| \geq 2n + 8$

Hence, $rn^D(B_{n,n}) = n^2$ + 3n + 8 , n $\geq$ 2.

**Theorem 2.5**

The radio D-distance number of subdivision of a star graph $S(K_{1,n})$,

$rn^D(S(K_{1,n}))$ = 6n + 11, n $\geq$ 2

**Proof**

Let V($S(K_{1,n})$) = {$v_0, v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n$} and E($S(K_{1,n})$) = {$v_0 v_i$, $v_i u_i$},1 $\leq i \leq$ n, where $v_0$ is the apex vertex. Then $d^D(v_i, u_i)$ = 4, 1$\leq i \leq$ n, $d^D(v_0, u_i)$ = n + 5, 1$\leq i \leq$ n,

$d^D(v_i, v_j)$ = n + 6, 1$\leq i,j \leq$ n, $d^D(v_i, u_j)$ = n + 8, 1$\leq i \leq$ n, 1$\leq j \leq$ n, $i \neq j$.

$d^D(u_i, u_j)$ = n + 10, 1$\leq i \leq$ n, 1$\leq j \leq$ n, $i \neq j$. So $diam^D$(S($K_{1,n}$)) = n + 10.

Let f($u_1$) < f($u_2$) < f($u_3$) < . . . < f($u_{n-1}$) < f($u_n$) < f($v_1$) < f($v_2$) < . . . < f($v_n$) < f($v_0$).

The radio D-distance condition becomes

$d^D(u_1, u_2)$ + | f($u_1$) – f($u_2$) | ≥ n + 11

$d^D(u_2, u_3)$ + | f($u_2$) – f($u_3$) | ≥ n + 11, f($u_i$) = i, 1$\leq i \leq$ n, f($u_n$) = n

$d^D(u_n, v_1)$ + | f($u_n$) – f($v_1$) | ≥ n + 11,

and $d^D(u_1, v_1)$ + | f($u_1$) – f($v_1$) | ≥ n + 11, f($v_1$) = max{n +3, n + 8} = n + 8

and $d^D(v_1, v_2)$ + | f($v_1$) – f($v_2$) | ≥ n + 11,

f($v_i$) = n + 5i + 3, 1$\leq i \leq$ n.

$d^D(v_n, v_0)$ + | f($v_n$) – f($v_0$) | ≥ n + 11,f($v_0$) = 6n + 11

Hence, $rn^D$(S($K_{1,n}$)) = 6n + 11, n $\geq$ 2.

**Theorem 2.6**

The radio D – distance number of complete bipartite graph $K_{m,n}$ is

$rn^D(K_{m,n})$ = $n^2$ + m (2 – n) + n + 1, n $\geq$ m $\geq$ 2.

**Proof**

Let V($K_{m,n}$) = A ∪ B, where A = {$u_1, u_2, \ldots, u_m$} and B = {$v_1, v_2, \ldots, v_n$} be the partite sets. Then $d^D(u_i, v_j)$ = n + m + 1, 1$\leq i \leq$ m , 1$\leq j \leq$ n, $d^D(v_i, v_j)$ = n + 2m + 2, 1 $\leq i,j \leq$ n,

$d^D(u_i, u_j)$ = 2n + m + 2, 1 $\leq i,j \leq$ m. So $diam^D(K_{m,n})$ = 2n + m + 2, n $\geq$ m $\geq$ 2.

Then, by th radio D-distance condition f(A) = {1, 2, . . . , m}

Without loss of generality, let f($u_1$) < f($u_2$) < f($u_3$) < . . . < f($u_{m-1}$) < f($u_m$).

That is, f($u_m$) = m. And let f($u_m$) < f($v_1$) < f($v_2$) < . . . < f($v_{m-1}$) < f($v_m$).

$d^D(u_m, v_1)$ + | f($u_m$) – f($v_1$) | ≥ 2n + m + 3, f($v_1$) ≥ n + m + 2, f($v_1$) = n + m + 2

$d^D(v_1, v_2)$ + | f($v_1$) – f($v_2$) | ≥ 2n + m + 3,f($v_2$) = 2n + 3

$d^D(v_2, v_3)$ + | f($v_2$) – f($v_3$) | ≥ 2n + m + 3,f($v_3$) = 3n – m + 4

$d^D(v_3, v_4)$ + | f($v_3$) – f($v_4$) | ≥ 2n + m+ 3,f($v_4$) = 4n – 2m + 5

f($v_i$) = in – (i – 2)m + (i + 1) , 3 $\leq i \leq$ n,

Hence, $rn^D(K_{m,n})$ = $n^2$ + m (2 – n) + n + 1, n $\geq$ m $\geq$ 2.

**Note**

When m = n, $rn^D(K_{m,n})$ = 3n + 1.

**Definition**

The graph $C_n^{(t)}$ denoting the one point union of t copies cycle $C_n$. The graph $C_3^{(t)}$ ( or $K_3^{(t)}$ ) is called friendship graph.

**Theorem 2.7**

The radio D – distance number of friendship graph $C_3^{(t)}$ is

$rn^D(C_3^{(t)})$ = 3t + 6, t $\geq$ 2.

**Proof**

Let V($C_3^{(t)}$) = {$v_0, v_1, v_2, \ldots, v_t, v_{t+1}, v_{t+2}, \ldots, v_{2t}$}, where $v_0$ is the apex vertex.

Let E($C_3^{(t)}$) = {$v_0 v_i$, 1 $\leq i \leq$ 2t, $v_i v_{t+i}$, 1 $\leq i \leq$ t}. Then $d^D(v_i, v_{t+i})$ = 5, 1 $\leq i \leq$ t,

$d^D(v_0, v_i)$ = 2t+3, 1 $\leq i \leq$ 2t, $d^D(v_{t+i}, v_{i+1})$ = $d^D(v_{t+i}, v_{t+i+1})$ = 2t + 6, 1 $\leq i \leq$ t .

So $diam^D(C_3^{(t)})$ = 2t + 6. Let f($v_1$) < f($v_2$) < f($v_3$) < . . . < f($v_t$) < f($v_{t+1}$) < . . . < f($v_{t+(t-1)}$) < f($v_{2t}$) < f($v_0$).

The radio D-distance condition becomes

$d^D(v_i, v_{i+1})$ + | f($v_i$) – f($v_{i+1}$) | ≥ 2t + 7, f($v_i$) = i, 1 $\leq i \leq$ t

$d^D(v_t, v_{t+1})$ + | f($v_t$) – f($v_{t+1}$) | ≥ 2t + 7,f($v_{t+1}$) = t + 1

But, $d^D(v_1, v_{t+1})$ + | f($v_1$) – f($v_{t+1}$) | ≥ 2t + 7, f($v_{t+1}$) = 2t + 3 and

$d^D(v_1, v_{t+1})$ + | f($v_1$) – f($v_{t+1}$) | ≥ 2t + 7,f($v_{t+1}$) = 2t + 3

$d^D(v_2, v_{t+2})$ + | f($v_2$) – f($v_{t+2}$) | ≥ 2t + 7, f($v_{t+2}$) = 2t + 4, f($v_{t+i}$) = 2t + i + 2, 1 ≤ $i$ ≤ t

$d^D(v_{2t}, v_0)$ + | f($v_{2t}$) – f($v_0$) | ≥ 2t + 7, f($v_0$) = 3t + 6. Therefore, $rn^D(C_3^{(t)})$ ≤ 3t + 6, t ≥ 2

Therefore, $rn^D(C_3^{(t)})$ = 3t + 6, t ≥ 2.

**Theorem 2.8**

The radio D – distance number of degree splitting of a bistar graph DS($B_{n,n}$) is

$rn^D$ ( DS($B_{n,n}$)) $= 4n^2$ + 5n + 9, n ≥ 2.

**Proof**

Let V(DS( $B_{n,n}$)) = {u,v,$u_i$,$v_i$,$w_1$,$w_2$ : 1 ≤ i ≤ n} and E(DS( $B_{n,n}$)) = {uv, $uu_i$, $vv_i$, $u_iw_1$, $v_iw_1$, $uw_2$, $vw_2$ : 1 ≤ i ≤ n}. Then, $d^D(u_i, w_2)$ = n + 8, 1 ≤ $i$ ≤ n, $d^D$(u, v) = 2n + 5,

$d^D(u_i, v)$ = $d^D$(u, $v_i$) = 2n + 8, 1 ≤ $i$ ≤ n, $d^D(u_i, w_1)$ = $d^D(w_1, v_i)$ = 2n + 3, 1 ≤ $i$ ≤ n

$d^D(w_1, w_2)$ = 3n + 9, So $diam^D$ (DS($B_{(n,n)}$)) = 3n + 9, n ≥ 2.

Let

f($w_1$) < f($w_2$) < f($u_1$) < . . . < f($u_{n-1}$) < f($u_n$) < f(v) < $f(u)$ < f($v_1$) < f($v_2$) < . . . < f($v_n$).

The radio D-distance condition becomes, $d^D(w_1, w_2)$ + | f($w_1$) – f($w_2$) | ≥ 3n + 10

$d^D(w_2, u_1)$ + | f($w_2$) – f($u_1$) | ≥ 3n + 10, f($u_1$) = 2n + 4

$d^D(u_1, u_2)$ + | f($u_1$) – f($u_2$) | ≥ 3n + 10, f($u_i$) = 2in + 2i + 2, 1 ≤ $i$ ≤ n

f($u_n$) = $2n^2$ + 2n + 2, $d^D(u_n, v)$ + | f($u_n$) – f(v) | ≥ 3n + 10, f(v) = $2n^2$ + 3n + 4

$d^D$(v, u) + | f(v) – f(u) | ≥ 3n + 10, f(u) = $2n^2$ + 4n + 9

$d^D(u, v_1)$ + | f(u) – f($v_1$) | ≥ 3n + 10, f($v_1$) = $2n^2$ + 5n + 11

$d^D(v_1, v_2)$ + | f($v_1$) – f($v_2$) | ≥ 3n + 10, f($v_2$) = $2n^2$ + 7n + 13

$d^D(v_2, v_3)$ + | f($v_2$) – f($v_3$) | ≥ 3n + 10, f($v_3$) = $2n^2$ + 9n + 15

f($v_i$) = $2n^2$ + (2in + 3)n + 2i + 9, 1 ≤ $i$ ≤ n.

Therefore, $rn^D$ ( DS($B_{(n,n)}$)) = $4n^2$ +5n + 9, n ≥ 2

**Theorem 2.9**

The radio D-distance number of splitting of a star graph S'($K_{1,n}$) is

$rn^D$( S'($K_{1,n}$)) $= 3n^2$ + 3n + 6, n ≥ 2.

**Proof**

Let V($S'(K_{1,n})$) = {$u_1$, $u_2$, . . . , $u_n$,$v_1$, $v_2$, . . . , $v_n$,$w_1$, $w_2$} and

E($S'(K_{1,n})$) = {$u_iw_1$, $w_1v_i$, $v_iw_2$, 1 ≤ $i$ ≤ n }.Then $d^D$($u_i$, $w_2$) = 3n + 6, 1 ≤ $i$ ≤ n,

$d^D(u_i, u_j)$ = 2n + 4, 1 ≤ $i, j$ ≤ n, $i \neq j$, $d^D$($u_i$,$w_1$) = 2n + 2, 1 ≤ $i$ ≤ n, $d^D(w_1, v_i)$ = 2n + 3, 1 ≤ $i$ ≤ n, $d^D(v_i, u_j)$ = n + 6, 1 ≤ $i, j$ ≤ n, $i \neq j$, $d^D$($w_1$,$w_2$) = 3n + 4 .

So $diam^D$ (S'($K_{1,n}$)) = 3n + 6, n ≥ 2. Let f($u_1$) < f($w_2$) < f($u_2$) < f($u_3$) < . . . < f($u_n$) < f($w_1$) < f($v_1$) < f($v_2$) < . . . < f($v_n$).

The radio D-distance condition is $d^D(u_1, w_2)$ + | f($u_1$) – f($w_2$) | ≥ 3n + 7,

$d^D(w_2, u_2)$ + | f($w_2$) – f($u_2$) | ≥ 3n + 7,

$d^D(u_1, u_2)$ + | f($u_1$) – f($u_2$) | ≥ 3n + 7,

$d^D(u_2, u_3)$ + | f($u_2$) – f($v_3$) | ≥ 3n + 7,

f($u_i$) = (i – 1)n + 3i – 2, 2 ≤ $i$ ≤ n. $d^D(u_n, w_1)$ + | f($u_n$) – f($w_1$) | ≥ 3n + 7,

$d^D(w_1, v_1)$ + | f($w_1$) – f($v_1$) | ≥ 3n + 7, $d^D(v_1, v_2)$ + | f($v_1$) – f($v_2$) | ≥ 3n + 7

$d^D(v_2, v_3)$ + | f($v_2$) – f($v_3$) | ≥ 3n + 7,

f($v_i$) = $n^2$ + (2i + 2)n + (i + 6), 1 ≤ $i$ ≤ n.

Therefore, $rn^D$ ($S'(K_{1,n})$) = $3n^2$ + 3n + 6 , n ≥ 2.

**Theorem 2.10**

The radio D-distance number of Book graph $K_2$ + n$K_2$ (or $B_4^n$), $rn^D$($K_2$ + n$K_2$) = $2n^2$ – 6n + 1 if n ≥ 5.

**Proof**

Let V($K_2$ + n$K_2$) = {u, v, $u_i$, $v_i$ : 1 ≤ $i$ ≤ n} be vertex set. Let V($K_2$ + n$K_2$) = {uv, $uu_i$, $vv_i$, $u_iv_i$ : 1 ≤ $i$ ≤ n} be edge set. Then $d^D$(u, $v_i$) = n + 7, 1 ≤ $i$ ≤ n and $d^D$(u, v) = 2n + 3, $d^D(u, u_i)$ = n +4, $d^D$($u_i$, $u_{i+1}$) = n +7, $d^D$($u_i$, $v_i$) = 5, 1 ≤ $i$ ≤ n $d^D(v_1, u_n)$ = n + 10, , $d^D(v, u_i)$ = n + 7 So $diam^D$ ($K_2$ + n$K_2$) = 2n + 3.

Let f($u$) < f($v$) < f($u_1$) < f($u_2$) < f($u_3$) < . . . < f($u_n$) < f($v_1$) < f($v_2$) < . . . < f($v_n$).

The radio D-distance condition becomes

$d^D$(u, v) + $|f(u) - f(v)|$ ≥ 2n + 4,

$d^D$(v, $u_1$) + $|f(v) - f(u_1)|$ ≥ 2n + 4, But, $d^D(u, u_1) + |f(u) - f(u_1)| \geq$ 2n + 4,

So, $f(u_1)$ = max{n – 1, n + 1}= n + 1, $f(u_1)$ = n + 1. $d^D(u_1, u_2) + |f(u_1) - f(u_2)| \geq$ 2n + 4 $d^D(u, u_2) + |f(u) - f(u_2)| \geq$ 2n + 4,

$f(u_i)$ = in – (3i – 4), $1 \leq i \leq$ n. $d^D(u_n, v_1) + |f(u_n) - f(v_1)| \geq$ 2n + 4,

But, $d^D(v, v_1) + |f(v) - f(v_1)| \geq$ 2n + 4,

And $d^D(u_1, v_1) + |f(u_1) - f(v_1)| \geq$ 2n + 4,

$f(v_i) = n^2 + (i-3)n - 3i + 1$, $1 \leq i \leq$ n.

Therefore, $rn^D(K_2 + nK_2) = 2n^2 - 6n + 1$, n ≥ 5.

**Theorem 2.11**

The radio D – distance number of splitting of a bistar graph S'($B_{n,n}$) is

$rn^D(S'(B_{n,n})) = 6n^2 + 16n + 18$, $n \geq 2$.

**Proof**

Let V( S'($B_{n,n}$)) = {$u_i, v_i, u'_i, v'_i$, u, $v, u', v'$ : $1 \leq i \leq$ n } and E(S'($B_{n,n}$)) = {$u_iu'$, $u'v, u_iu$, $uu'_i$, $uv'$, $uv$, $vv'_i$, $vu_i$, $v_iv'$, $1 \leq i \leq$ n } .Then $d^D(u'_i, u') =$ 3n + 9, $1 \leq i \leq$ n,

$d^D(u'_i, v_i)$ = 3n + 9, $1 \leq i \leq$ n, $d^D(u_i, v_i)$ = 3n + 10, $1 \leq i \leq$ n, $d^D(v_i, v_j)$ = n + 7, $1 \leq i, j \leq$ n,

$d^D(v'_i, v)$ = 2n + 4, $1 \leq i \leq$ n, $d^D(v, v')$ = 3n + 7, $d^D(v_i, v'_i)$ = 2n + 7, $1 \leq i \leq$ n,

$d^D(v'_i, v'_j)$ = 2n + 6, $1 \leq i, j \leq$ n, $d^D(v', u)$ = 3n + 4. So $diam^D$(S'($B_{n,n}$)) = 3n + 10, $n \geq 2$.

Let, $f(u_1) < f(v_1) < f(v_2) < \ldots < f(v_n) < f(v'_1) < f(v'_2) < \ldots < f(v'_n) < f(v) < f(v') <$

$f(u) < f(u'_1) < f(u'_2) < \ldots < f(u'_n) < f(u') < f(u_2) < f(u_3) < f(u_n)$.

The radio D-distance condition becomes

$d^D(u,v) + |f(u) - f(v)| \geq diam^D$(S'($B_{n,n}$)) + 1. Now, $d^D(u_1, v_1) + |f(u_1) - f(v_1)| \geq$ 3n + 11 $d^D(v_1, v_2) + |f(v_1) - f(v_2)| \geq$ 3n + 1,

$f(v_i)$ = 2(i – 1)n + 4(i - 1) + 2, $2 \leq i \leq$ n

$d^D(v_n, v'_1) + |f(v_n) - f(v'_1)| \geq$ 3n + 11,

$d^D(v'_1, v'_2) + |f(v'_1) - f(v'_2)| \geq$ 3n + 11,

$f(v'_i) = 2n^2$ + (i + 2)n + 5i – 3, 1 ≤ i ≤ n.

$d^D(v'_n, v) + |f(v'_n) - f(v)| \geq$ 3n + 11,

$d^D(v, v') + |f(v) - f(v')| \geq$ 3n + 11

$d^D(v', u) + |f(v') - f(u)| \geq$ 3n + 11,

$d^D(u, u'_1) + |f(u) - f(u'_1)| \geq$ 3n + 11,

$d^D(u'_1, u'_2) + |f(u'_1) - f(u'_2)| \geq$ 3n + 11

$f(u'_i) = 3n^2$ + (i + 8)n +5i + 17, 1 ≤ i ≤ n.

$d^D(u'_n, u') + |f(u'_n) - f(u')| \geq$ 3n + 11,

$d^D(u', u_2) + |f(u') - f(u_2)| \geq$ 3n + 11

$d^D(u_2, u_3) + |f(u_2) - f(u_3)| \geq$ 3n + 11

$f(u_i) = 4n^2$ + (2i + 11)n + 4i + 18, 1 ≤ i ≤ n.

Therefore, $rn^D(S'(B_{n,n})) = 6n^2 + 15n + 18$ , $n \geq 2$.

**Theorem 2.12**

The radio D-distance number of triangular snake $TS_n$ is $rn^D(TS_n) = 10n^2$ – 47n + 60

**Proof**

Let $V(TS_n) = \{v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_{n-1}\}$ and $E(TS_n) = \{v_iu_i, u_iv_{i+1}, v_iv_{i+1} : 1 \leq i \leq$ n – 1}. Then, $d^D(v_1, v_n) = d^D(v_1, u_{n-1})$ = 5n – 5, $d^D(v_1, v_{n-1})$ = 5n – 8

$d^D(v_1, v_2) = d^D(v_{n-1}, v_n) = d^D(u_i, v_{i+1})$ = 7, : $1 \leq i \leq$ n – 2, $d^D(v_1, u_1) = d^D(v_n, u_{n-1})$ = 5

So, $diam^D(TS_n)$ = 5n – 5. The radio D-distance condition becomes

$d^D(u,v) + |f(u) - f(v)| \geq diam^D(TS_n)$ + 1

Let, $f(v_1) < f(v_n) < f(v_2) < f(v_3) < \ldots < f(v_{n-1}) < f(u_1) < f(u_2) < \ldots < f(u_{n-1})$.

Now,

$d^D(v_1, v_n) + |f(v_1) - f(v_n)| \geq$ 5n – 4

$d^D(v_n, v_2) + |f(v_n) - f(v_2)| \geq$ 5n – 4

But, $d^D(v_1, v_2) + |f(v_1) - f(v_2)| \geq$ 5n – 4

$d^D(v_2, v_3) + |f(v_2) - f(v_3)| \geq$ 5n – 4

$f(v_i)$ = 5(i – 1)n – 13i + 16, 1 ≤ i ≤ n – 1

$d^D(v_{n-1}, u_1) + |f(v_{n-1}) - f(u_1)| \geq$ 5n – 4

But, $d^D(v_1, u_1) + |f(v_1) - f(u_1)| \geq$ 5n – 4

Also, $d^D(v_2, u_1) + |f(v_2) - f(u_1)| \geq 5n - 4$

So, $f(u_1) = \max\{5n^2 - 23n + 32, 5n - 8, 10n - 21\}$

$d^D(u_1, u_2) + |f(u_1) - f(u_2)| \geq 5n - 4$

$d^D(u_2, u_3) + |f(u_2) - f(u_3)| \geq 5n - 4$

$f(u_i) = 5n^2 - (5i - 28)n - 14i + 46, 1 \leq i \leq n - 1.$

Therefore,

$rn^D(TS_n)) = 10n^2 - 47n + 60, \; n \geq 4.$

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