Thermal Stress Analysis Of An Isotropic And Single Layered Laminated Orthotropic Plate

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***Abstract*— Analytical models in an isotropic and single layered laminated orthotropic plate gives shear deformation and transverse normal thermal strains is validated for the thermal stress analysis subjected to gradient thermal profile across the thickness of laminate. Benchmark results for isotropic are prepared for sinusoidal and parabolic thermal profiles. First Order Shear Deformation Theory is used for calculating various quantities. The paper consists of results which give good agreement with gradient, sinusoidal and parabolic thermal profiles especially for thin plates having aspect ratio more than 10.**

***Keywords*— Isotropic, orthotropic, Laminates, Stress analysis, Thermal, Deformation**

I. INTRODUCTION

With the advancement of the technology of laminated materials, it is now possible to use these materials in high temperature situations. However, composites have no yield-limit, unlike metals and have a variety of failure modes, such as fiber failure, matrix cracking, interfiber failure and delamination, which give rise to a damage growing in service.

Moreover, composite plates are subjected to significant thermal stresses due to different thermal properties of the adjacent laminas and therefore accurate predictions of thermally induced deformations and stresses represent a major concern in design of conventional structures.

Behavior of composite plates can be characterized by a complex 3D state of stress. In many instances, these laminated structural elements are moderately thick in relation to their span dimensions. For thick or moderately thick structural elements, the normal to the mid surface is distorted due to in homogeneity in the transverse shear moduli, which is smaller than in-plane Young's moduli, resulting in significant.

Various theories and models are reported for the thermal stress analyses of the laminates. In this article First Order Shear Deformation Theory is used for calculating the results.

Most of the researchers assumed linear gradient or constant thermal profiles along the thickness of plates. Isotropic and Single layered orthotropic square plates are assumed in this paper for preparation of benchmark results. Material properties are shown in Table 1.1 and Table 1.2

The prime objective of this paper is to suggest the parabolic and sinusoidal thermal profiles across thickness of the plate which is shown in Table 2. This is suggested along with the gradient profile and analytical solutions along with bench mark results in normalization form are prepared.

TABLE I. MATERIAL PROPERTIES FOR CASE 1

Square Plate $a = b$	Isotropic plate
Material Properties	Ceramic (Alumina Al_2O_3): Youngs Modulus: E_C =380GPa; Poisons ratio: ν_c =0.3; Coeff. of thermal expansion: α_c =7.4x10^{-6} /k
Normalization	$\bar{u} = \frac{u}{\alpha_c T_0 h}, \bar{w} = \frac{w}{\alpha_c T_0 h}, \bar{\sigma}_x = \frac{\sigma_x}{\alpha_c T_0 E_c}, \bar{\tau}_{xy} = \frac{\tau_{xy}}{\alpha_c T_0 E_c}.$
Reference	Matsunaga [6]
Maximum normalized stress coordinates; $\bar{\sigma}_x$: *(a/2, b/2, ±h/2);* $\bar{\tau}_{xy}$ *(0, 0, ±h/2).* For analysis positive coordinates considered.	
Maximum displacement coordinates, $\bar{u}$ *: (0, b/2, ±h/2);* $\bar{w}$ *: (a/2, b/2,0)*	

TABLE II. MATERIAL PROPERTIES FOR CASE 1

Square Plate $a = b$	Single layer orthotropic plate
Material Properties	E_1 =150 GPa, E_2 = 10GPa, $\nu_{12} = 0.3,\ \nu_{21} = 0.02$, $G_{12} = G_{13} = 5GPa$, $G_{23} = 3.378GPa$, $\alpha_x = 0.139E\text{-}6/\text{k}$, $\alpha_y = 9E\text{-}6/\text{k}$
Normalization	$\bar{u} = \frac{u}{\alpha_1 T_0 h S^{\wedge}3}$, $\bar{v} = \frac{v}{\alpha_1 T_0 h S^{\wedge}3}$, $\bar{w} = \frac{w}{\alpha_1 T_0 h S^{\wedge}3}$, $\bar{\sigma}_x = \frac{\sigma_x}{\alpha_1 T_0 E_2 S^{\wedge}2}$, $\bar{\sigma}_y = \frac{\sigma_y}{\alpha_1 T_0 E_2 s^{\wedge}2}$, $\bar{\tau}_{xy} = \frac{\tau_{xy}}{\alpha_1 T_0 E_2 S^{\wedge}2}$. $\bar{\tau}_{xz} = \frac{\tau_{xz}}{\alpha_1 T_0 E_2 S}$, $\bar{\tau}_{yz} = \frac{\tau_{yz}}{\alpha_1 T_0 E_2 S}$
Reference	T.Kant et al. [5]
Maximum normalized stress coordinates; $\bar{\sigma}_x$ and $\bar{\sigma}_y$ *(a/2, b/2, ±h/2);* $\bar{\tau}_{xy}$ *(0, 0, ±h/2).* For analysis positive coordinates considered.	
Maximum displacement coordinates, $\bar{u}$ *: (0, b/2, ±h/2);* $\bar{v}$ *: (a/2, 0, ±h/2);* $\bar{w}$ *: (a/2, b/2,0)*	
E_1 and E_2 : Modulus of Elasticity along laminate direction (x-axis) and transverse direction (y-axis). ν_{12} and ν_{21} : poisons ratio. G_{12} , G_{13} , G_{23} : Modulus of rigidity. α_x , α_y : Thermal coefficients along x and y axes. S= a/h: aspect ratio. a, b, h: Dimensions of plate along x, y and z directions.	

TABLE III. PROPOSED THERMAL LOAD PROFILES ALONG THICKNESS OF PLATES BOUNDARY CONDITIONS AT (A/2, B/2,± H/2) ARE

$$-T_0 \le T \le T_0$$

Thermal Profile	Equation of profile
TP1	$T = \frac{2z}{h} T_0 \sin(\pi x/a)\sin(\pi y/b)$ (Gradient)
TP2	$T = (2z/h)^3 T_0 \sin(\pi x/a)\sin(\pi y/b)$ (Cubic parabola)
TP3	$T = \sin(\pi z/h) T_0 \sin(\pi x/a)\sin(\pi y/b)$ (Sinusoidal)

Earlier research based on First Order Shear Deformation Theory (FOST) considered in Reissner, [1] and Mindlin [2] for solutions to include the thermal effects on laminates. Deficiencies in the FOST are Rower et al.,[3] removed the by incorporating third and fifth order displacement approximations through the plate thickness. 3D elasticity solution can estimate the correct results of the thermally induced quantities like displacements and stresses. Kant and Swaminathan [4] presented simplified formulations through the paper Analytical solutions for static analysis of laminated composite and sandwich plates based on a higher order refined theory and suggested First Order Shear Deformation Theory as a special case with its importance. Kant et al.[5] developed semi-analytical solution for constant and linear temperature variation through the thickness of a laminate for composites and sandwiches. Matsunaga [6] prepared basic transeverse displacements for istropic plate. Kant and Shiyekar [7] developed higher order theory for composite laminates subjected to thermal gradient using (HOSNT12) model.

II. FORMULATIONS

A. Displacement model

A simply supported single layer orthotropic laminated plate is presented along with complete analytical solution. The geometry of the laminate is such that the side '*a*' is along '*x*' axis and side '*b*' is on '*y*' axis. The thickness of the laminate is denoted by '*h*' and is coinciding with '*z*' axis. The reference mid-plane of the laminate is at *h/2* from top or bottom surface of the laminate as shown in the Figure 1. Lamina axes and reference axes are coincides with each other. The formulation is assuming fiber direction of the single layered lamina is coinciding with '*x*' axis of the laminate. Figure also illustrates the mid-plane positive set of displacements along *x-y-z* axes.

u (x,y,z) , *v (x,y,z)* , *w (x,y,z)* are the displacements in *x, y, z* directions respectively, can be written as -

$$u = z\theta_x \cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right),$$
$$v = z\theta_y \sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right), \qquad (2.1)$$
$$w = w_0 \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)$$

They θ_x, θ_y are the rotation of the normal to the middle-plane about *y* and *x*-axes respectively. Here for this article, m=n=1 is considered for FOST.

B. Constitutive Relationship

Stress-strain relationship for orthotropic laminate under linear thermal loading can be written as – Stress-strain relation for 0^0 layer.

$$\begin{Bmatrix}\sigma_{x_0}\\ \sigma_{y_0}\\ \tau_{xy_0}\end{Bmatrix} = \begin{bmatrix} Q_{11_0} & Q_{12_0} & 0\\ Q_{21_0} & Q_{22_0} & 0\\ 0 & 0 & Q_{33_0}\end{bmatrix}\begin{Bmatrix}\varepsilon_x - \alpha_x T\\ \varepsilon_y - \alpha_y T\\ \gamma_{xy}\end{Bmatrix}$$
$$\begin{Bmatrix}\tau_{xz_0}\\ \tau_{yz_0}\end{Bmatrix} = \begin{bmatrix} Q_{44_0} & 0\\ 0 & Q_{55_0}\end{bmatrix}\begin{Bmatrix}\gamma_{xz}\\ \gamma_{yz}\end{Bmatrix} \qquad (2.2)$$

Where $(\sigma_x, \sigma_y, \tau_{xy}, \tau_{yz}, \tau_{xz})$ are the stresses and $(\varepsilon_x, \varepsilon_y, \gamma_{xy}, \gamma_{yz}, \gamma_{xz})$ are the strains with respect to laminate coordinate system (*x-y-z*).

$$\varepsilon_x = \frac{\partial u}{\partial x}, \quad \varepsilon_y = \frac{\partial v}{\partial y}, \quad \varepsilon_z = \frac{\partial w}{\partial z}, \tag{2.3}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}$$

$[Q_{ij}]$ are transformed elastic constants or stiffness matrix and defined as per the following.

$$Q_{11_0} = \frac{E_1}{1-\nu_{12}\nu_{21}}, Q_{12_0} = \frac{\nu_{12}E_1}{1-\nu_{12}\nu_{21}},$$

$$Q_{22_0} = \frac{E_2}{1-\nu_{12}\nu_{21}}, Q_{33_0} = Q_{33_{90}} = G_{12},$$

$$Q_{44_0} = G_{13}, Q_{55_0} = G_{23}, \frac{\nu_{21}}{E_2} = \frac{\nu_{12}}{E_1}. \tag{2.4}$$

$$Q_{11_{90}} = \frac{E_2}{1-\nu_{12}\nu_{21}}, Q_{12_{90}} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}},$$

$$Q_{21_{90}} = Q_{12_{90}}, Q_{22_{90}} = \frac{E_1}{1-\nu_{12}\nu_{21}},$$

$$Q_{44_{90}} = G_{23}, Q_{55_0} = G_{13}$$

$\alpha_x, \alpha_y =$ Coefficient of thermal expansion in x, y direction, T is temperature profile along thickness direction as shown in Table 1. As stated in eq. (4), the compliance matrix involves engineering properties namely two extensional modulii (E_1, E_2), two Poisson's ratios (ν_{12}, ν_{21}) and three shear modulii (G_{12}, G_{13}, G_{23}).

For isotropic plate

$$\nu_c = \nu_{12} = \nu_{21}, \alpha_x, \alpha_y = \alpha_C, E_C = E_1 = E_2,$$

$$G_{12} = G_{13} = G_{23} = G, \tau_{yz} = \tau_{xz} = 0$$

III. Discussion

CASE 1. Isotropic plate

A discussion on the numerical study of simply supported symmetrical isotropic plate subjected to 3 thermal loading *(a = b)* under thermal load profiles 1 to 3 for different aspect ratios (a/h = 2, 5, 10, 20, 50, 100). Table 3 and 4 shows the comparison of results of normalized transverse displacement $\bar{w}$ between present First order shear deformation plate theory (FOST) with results presented by Mastunaga [6] and it is observed that thermal profile 1 gives close agreement with Mastunaga [6] hence for further comparison thermal profile 1 gives basis for comparing the other normalized terms namely in plane normal stresses $\bar{\sigma}_x$, transverse shear stress $\bar{\tau}_{xy}$ in-plane displacement $\bar{u}$.

TABLE IV. Benchmark Results For Transverse Displacement $\bar{w}$

S = a/h	Mastunaga [6]	Thermal Profile 1	Thermal Profile 2	Thermal Profile 3
2	0.4571	0.5269	0.3161	0.6406
5	3.227	3.2929	1.9758	4.0037
10	13.11	13.1718	7.9031	16.0149
20	52.62	52.687	31.6122	64.0597
50	-	329.2938	197.5763	400.3733
100	1317	1.32E+03	790.3052	1.60E+03

TABLE V. Benchmark Result Solutions Of % Error Transverse Displacement $\bar{w}$

S = a/h	Thermal Profile 1	Thermal Profile 2	Thermal Profile 3
5	2.04	-38.77	24.06
10	0.004	-39.71	22.15
20	0.12	-39.92	21.74
% Error in $\bar{w}$ = {Present TP –Matsunaga [6] / Matsunaga [6] } × 100.			

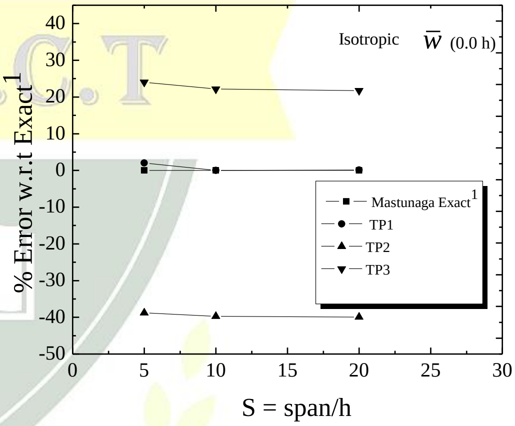

Fig. 1. %Variation of solutions of normalized transverse displacement $\bar{w}$

The table IV contains benchmark results solutions of normalized transverse displacements ($\bar{w}$) and table V gives the % error for transverse displacement ($\bar{w}$) under thermal load profiles 1 to 3 for different aspect ratios (S = a/h = 5, 10, 20) with respect to Mastunaga [6] and it is observed that profile 1 gives close agreement with Mastunaga [6] having % error 2.04, 0.004 and 0.12 for aspect ratios 5, 10 and 20 respectively. Profile 2 and 3 having huge amount of % error as compared with profile 1

Fig.1 shows variations and the % error variations of transverse displacement ($\bar{w}$) for comparison between present (FOST) and Mastunaga [6], for the aspect ratios (a/h= 5, 10, 20).

The table VI contains benchmark results solutions of in plane stresses $\bar{\sigma}_x$ under thermal load profiles 1 to 3 for different aspect ratios (a/h = 2, 5, 10, 20, 50, 100).

TABLE VI. Benchmark Results For Inplane Stresses $\bar{\sigma}_x$

S = a/h	Thermal Profile 1	Thermal Profile 2	Thermal Profile 3
2	-0.5	-21.7857	-7.4891
5	-0.5	-3.4857	-1.1983
10	-0.5	-0.8714	-0.2996
20	-0.5	-0.2179	-0.0749
50	-0.5	-0.0349	-0.012
100	-0.5	-0.0087	-0.003

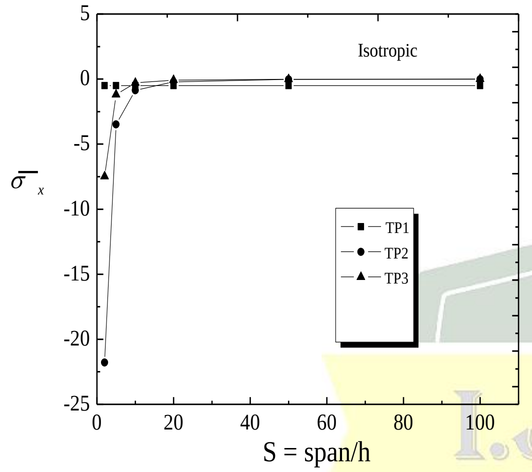

Fig. 2. Variation of normalized inplane stresses $\bar{\sigma}_x$.

Fig. 2 shows variation of results of normalized in plane normal stresses $\bar{\sigma}_x$ for aspect ratios 2, 5, 10, 20, 50 and 100 for thermal load profiles 1 to 3 and it is observed that graph of profile 1 gives close agreement with Mastunaga [6] while profile 2 and 3 graphs away from profile 1.

TABLE VII. BENCHMARK RESULTS FOR INPLANE SHEAR STRESSES $\bar{\tau}_{xy}$

S = a/h	Thermal Profile 1	Thermal Profile 2	Thermal Profile 3
2	-0.5	-0.75	-15.1982
5	-0.5	-0.12	-2.4317
10	-0.5	-0.03	-0.6079
20	-0.5	-0.0075	-0.152
50	-0.5	-0.0012	-0.0243
100	-0.5	-3.00E-04	-0.0061

The table VII contains benchmark results solutions of transverse shear stresses $\bar{\tau}_{xy}$ under thermal load profiles 1 to 3 for different aspect ratios (a/h = 2, 5, 10, 20, 50, 100).

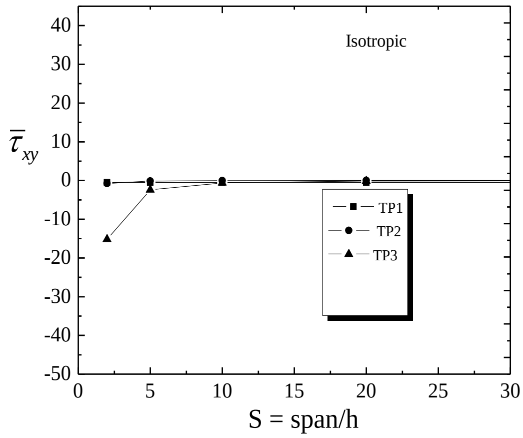

Fig. 3. Variation of normalized inplane shear stresses $\bar{\tau}_{xy}$.

Figure 3 shows variation of results solutions of normalized transverse shear stresses $\bar{\tau}_{xy}$ for aspect ratios 2, 5, 10, 20, 50, and 100 to 3 thermal loading.

TABLE VIII. BENCHMARK RESULTS FOR INPLANE DISPLACEMENT $\bar{u}$

S = a/h	Thermal Profile 1	Thermal Profile 2	Thermal Profile 3
2	-0.4138	-0.2483	-0.5031
5	-1.0345	-0.6207	-1.2578
10	-2.069	-1.2414	-2.5156
20	-4.138	-2.4828	-5.0312
50	-10.3451	-6.207	-12.5781
100	-20.6901	-12.4141	-25.1562

The tables VIII contains benchmark results solutions of normalized in plane displacements $\bar{u}$ under thermal load profiles 1 to 3 for different aspect ratios (a/h = 2, 5, 10, 20, 50, 100).

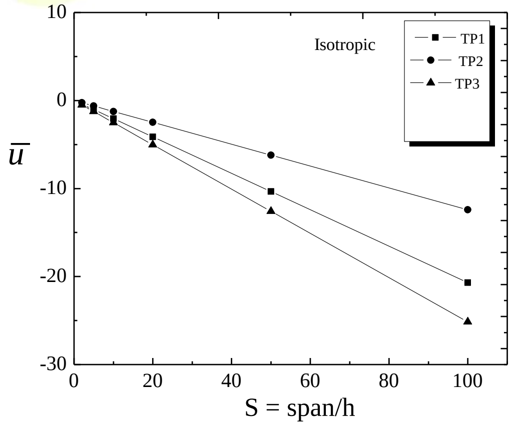

Fig. 4. Variation of normalized inplane displacements $\bar{u}$.

Figure 4 shows variation of results solutions of normalized in plane displacements $\bar{u}$ for aspect ratios for aspect ratios 2, 5, 10, 20, 50, and 100 to 3 thermal loading.

As the plate is isotropic benchmark results and graphs are similar for pairs ($\bar{\sigma}_x$, $\bar{\sigma}_y$) and ($\bar{u}$, $\bar{v}$).

CASE 2. Single layer orthotropic plate

A discussion on the numerical study of single layer 0^0 homogeneous orthotropic under liner thermal loading is presented. Comparison of results between present First order shear deformation plate theory (FOST) with Semi Analytical Model [SAM] presented by Kant et al. [5].

TABLE IX. TRANSVERSE DISPLACEMENTS 100 X $\bar{w}$ FOR DIFFERENT ASPECT RATIOS

S = a/h	Present FOST	Kant et al.[5]	% error
4	10.9618	9.0226	21.49
10	1.2226	1.4042	12.93
20	0.28	0.2916	4.32
% error in $\bar{w}$ = [Kant.[5]-Present FOST]x100/Kant.[5]			

TABLE X. INPLANE STRESSES $\bar{\sigma}_x$

S = a/h	Present FOST	Kant et al.[5]	% error
4	1.8105	2.0164	10.21
10	0.4389	0.4845	9.41
20	0.1189	0.1198	2.42
% error in $\bar{\sigma}_x$ = [Kant.[5]-Present FOST]x100/Kant.[5]			

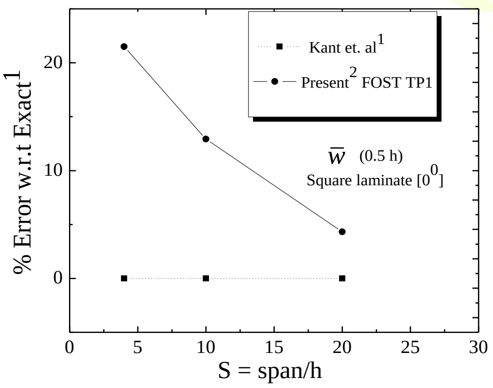

Fig. 5. % Variation of normalized transverse displacements $\bar{w}$.

The table II contains the material properties used for analysis. The table IV contains results of normalized transverse displacement ($\bar{w}$) and Estimation of % error. The % error for transverse displacement ($\bar{w}$) estimated around 21.49 % for (SAM) and (Present FOST) for plate (a/h =4) and 4.32 % for plate (a/h =20). Figures 6 shows the % error variations of transverse displacement ($\bar{w}$) for comparison between present (FOST) and (SAM) Kant et al. [5], through the aspect ratios (a/h= 4, 10, 20) of 0^0 homogeneous orthotropic simply supported square plate under linear thermal load (Thermal Profile 1).

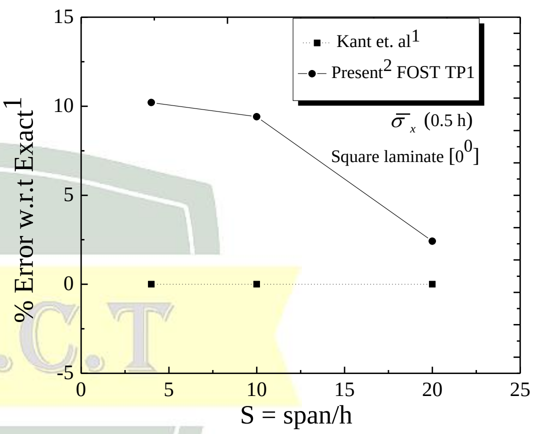

Fig. 6. % Variation of normalized inplane stresses $\bar{\sigma}_x$.

The table V contains results of normalized in plane stress ($\bar{\sigma}_x$) and Estimation of % error. The % error of normalized in plane stress ($\bar{\sigma}_x$) estimated around 10.21 % between (SAM) and (Present FOST) for plate (a/h =4) and 2.42 % for plate (a/h =20). Figure 6 shows variations and the % error in normalized in plane stress ($\bar{\sigma}_x$) for comparison between present (FOST) and (SAM) Kant et al. [5], through the aspect ratios (a/h= 4, 10, 20) of 0^0 homogeneous orthotropic simply supported square plate under linear thermal load.

TABLE XI. INPLANE STRESSES $\bar{\sigma}_y$

S = a/h	Present FOST	Kant et al.[5]	% error
4	-3.0284	-2.0164	47.45
10	-0.5636	-0.5638	0.03
20	-0.1447	-0.1448	0.07
% error in $\bar{\sigma}_y$ = [Kant.[5]-Present FOST]x100/Kant.[5]			

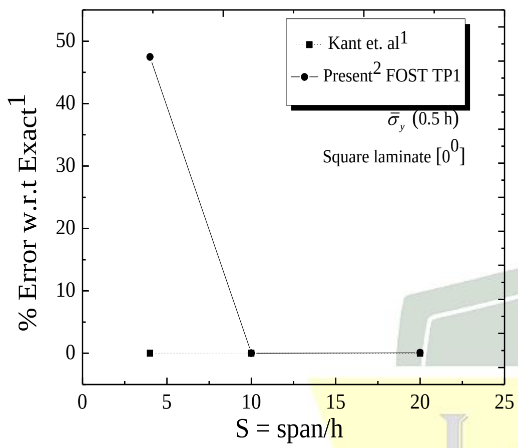

Fig. 7. % Variation of normalized inplane stresses $\bar{\sigma}_y$.

% Variation of normalized in plane stresses $\bar{\sigma}_y$ The table VI contains results of normalized in plane stress ($\bar{\sigma}_y$) and Estimation of % error. The % error of normalized in plane stress ($\bar{\sigma}_y$) estimated around 47.45 % between (SAM) and (Present FOST) for plate (a/h =4) and 0.07 for plate (a/h =20). Figure 8 respectively shows the % error in normalized in plane stress ($\bar{\sigma}_y$) for comparison between present (FOST) and (SAM) Kant et al.[5], through the aspect ratios (a/h= 4,10,20) of 0^0 homogeneous orthotropic simply supported square plate under linear thermal load.

The table VII contains results of normalized shear stress ($\bar{\tau}_{xy}$) and Estimation of % error. The % error of normalized shear stress ($\bar{\tau}_{xy}$) estimated around 72.34 % between (SAM) and (Present FOST) for plate (a/h =4) and 0.28 for plate (a/h =20). Figure 9 shows the % error in normalized shear stress ($\bar{\tau}_{xy}$) for comparison between present (FOST) and (SAM) Kant et al. [5], through the aspect ratios (a/h= 4, 10, 20) of 0^0 homogeneous orthotropic simply supported square plate under linear thermal load.

TABLE XII. INPLANE SHEAR STRESSES 10 X $\bar{\tau}_{xy}$

S = a/h	Present FOST	Kant et al.[5]	% error
4	-6.1296	-3.5566	72.34
10	-0.628	-0.638	1.56
20	-0.1401	-0.1405	0.28
% error in $\bar{\tau}_{xy}$ = [Kant.[5]-Present FOST]x100/Kant.[5]			

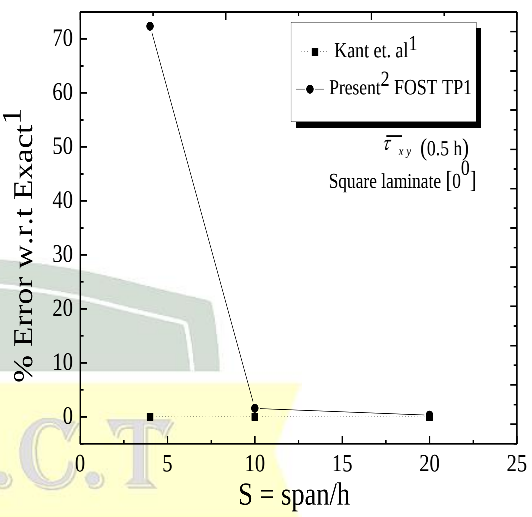

Fig. 8. % Variation of normalized inplane shear stresses $\bar{\tau}_{xy}$.

CONCLUSION

Isotropic plate under 3 thermal loadings and Single layered (0^0) laminated composite square plate under the thermal profiles 1 are analyzed. Following concluding remarks are marked.

- For Isotropic plate tthermal profile 1 gives comparatively good agreement in all the normalized quantities with values given by Mastunaga [6].
- FOST gives excellent results for thin plates in minimum efforts.
- With some factor of safety and appropriate shear correction factor, designing of thin plates can be recommended instead of complicated computer programs.
- For isotropic plate rresults are validated for some of the quantities hence bench mark results are prepared for thermal profile 1, 2 and 3.
- % error in Variation of transverse displacement $\bar{w}$, in plane stresses $\bar{\sigma}_x$, $\bar{\sigma}_y$ and transverse shear stress $\bar{\tau}_{xy}$ between present FOST and Higher order shear deformation theory [HOST] presented by Mastunaga.[6] are decreasing as a/h increasing means for thin plate. Results are almost matching specially for thermal profile 1 and matching % decreases for thermal profile 2 and 3 respectively.

- % error in Variation of transverse displacement $\bar{w}$, inplane stresses $\bar{\sigma}_x, \bar{\sigma}_y$ and transverse shear stress $\bar{\tau}_{xy}$ between present FOST and Semi analytical model [SAM] presented by Kant et al.[5] are decreasing as a/h increasing means for thin plate. Results are almost matching specially for thermal profile 1 and matching % decreases for thermal profile 2 and 3 respectively.

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