Flood frequency analysis for Tel sub-basin of Mahanadi River, India using Weibull, Gringorten and L-moments formula

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***Abstract*—The objective of this study is to determine the magnitude and frequency of floods for Tel sub-basin of Mahanadi river system, India using Gumbel distribution. The probability plot and flood-frequency curves by Gumbel distribution of each individual station are prepared using three different plotting position formulas (i.e. Weibull, Gringorten and L-moments). From the results and analysis of two stations of Tel sub-basin, Gumbel distribution based on L-moments always gives the least ratio of peak discharge of T years recurrence interval over mean annual flood (QT/MAF), also the pertinence of L-moments with Gumbel distribution have some limitations and it is only good for small samples data. If compared between Gumbel distribution by Weibull formula and Gumbel distribution by Gringorten formula, the latter is better because it gives the least ratio (which is in agreement with the literature). Therefore, it could be concluded that for both the stations L-moments method is the best, but since L-moments method have some limitations, Gringorten formula is still the best plotting position method to be applied with Gumbel distribution.**

Keywords— Gringorten; Gumbel Distribution; L-moment; Weibull

I. INTRODUCTION

In the planning and design of water resources projects, Engineers and Planners are often interested to determine the magnitude and frequency of floods that will occur at the project areas. Cunnane [1] had studied various plotting position methods using the criteria of unbiasedness and maximum variance. He found that the Weibull plotting position formula was biased, and it plotted the largest values of a sample at too small a return period. He said, for data distributed according to the Extreme Value Type I distribution (or Gumbel distribution), the Gringorten formula ($b = 0.44$) was the best. This paper focuses on the application of Gumbel distribution with Weibull formula, Gringorten formula and L-moments method. It is hoped that the findings from this study could subsidize to the acquaintance of the application of Gumbel distribution in flood-frequency analysis study in Tel sub-basin of Mahanadi river system, India. There are several types of theoretical probability distributions (or frequency distribution functions) that have been successfully applied to hydrologic data. Extreme Value Distribution which is further subdivided into three forms – EVI (Gumbel Distribution), EVII (Frechet Distribution) and EVIII (Weibull Distribution) [10]. In the United States and Australia the log Pearson Type III (LPIII) distribution has been selected as a standard by federal agencies [2]. The general extreme value (GEV) distribution is the standard method for flood-frequency analysis in the U.K. [10]. Parameters can be estimated from sample data using a range of procedures, including the methods of moments, maximum-likelihood and L-moments [11]. Gordon et al.[2] say, although no one distribution will fit all flood data, specifying the distribution used and the method of fitting it will allow other researchers to obtain some results from the same set of data.

II. STUDY AREA AND DATA COLLECTION

The Tel River originates in plain of Koraput district of Odisha, about 32 km to the west of Jorigam (Figure 1). It is the second largest river of Orissa and is an important tributary of the Mahanadi River. The river traverses a total length of 296 km to join the Mahanadi River on the right bank, 1.6 km below Sonepur. The total drainage area of the Tel River is about 22,818 km2, in which 11960 km2 lies up to Kesinga and 19600 km2 lies up to Kantamal gauging stations. The Tel sub-basin is bound between latitude 18° to 21° and between longitude 83° to 86° approximately. The normal annual rainfall of the entire Mahanadi basin is 1360 mm (16% coefficient of variation, CV) of which about 6%, i.e.1170 mm, occurs during the monsoon season (15 % CV) from June to September.

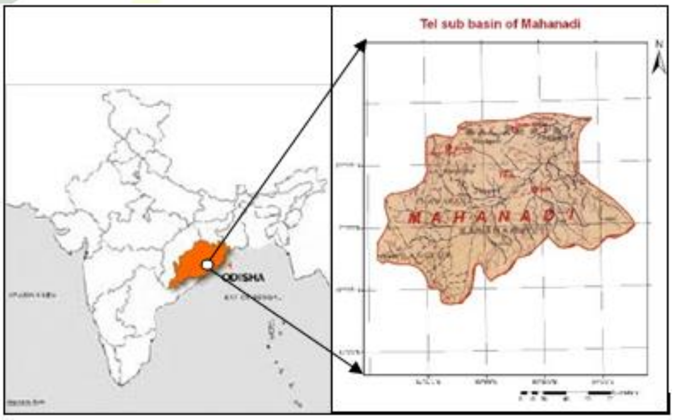


Fig. 1. General Location Map of the Tel sub-basin.

Daily discharge data for the years 1972-2009 were collected from Central Water Commission, Bhubaneswar. Figure 2 shows the daily mean discharge time series from 1972-2009 for Kantamal (downstream) and from 1979 to 2009 for Kesinga (upstream) station of Tel basin. In addition, a non-linear function was fitted to the time series, using locally weighted scatterplot smoothing (LOWESS). The LOWESS results illustrate that the series does not have major non-stationaries in frequency or variability.

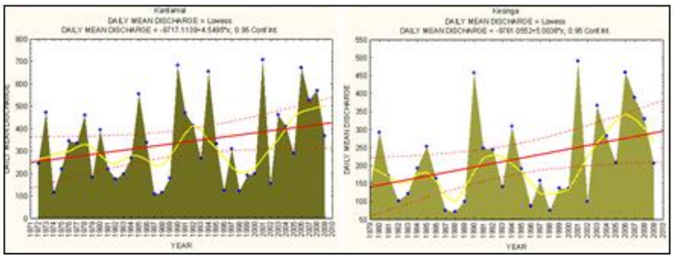


Fig. 2. Discharge time series including linear and non-linear trend of the Tel sub-basin, India.

III. Using the Template

A. Extreme Value Type I (also known as EVI or Gumbel Distribution)

EVI distribution (or Gumbel distribution) is a double exponential distribution. According to Ponce [5], the cumulative density function, F(x) of the Gumbel method is:

$$F(X)=e^{-e^{-y}} \qquad (1)$$

in which, F(x) is the probability of non exceedance. He added, in flood frequency analysis, the probability of interest is the probability of exceedance (i.e. the complementary probability to F(x)):

G(x) = 1 - F(x) (2)

Subramanya [6] also highlighted that in practice it is the value of X for a given P

yp= 1n [- 1n (1 - P)] (3)

Noting that T=1/P, he then noticed that by designating yT= the value of y, commonly called the reduced variate, for a given T, the following equations could be produced:

$$y_T=-\ln\left(\ln\frac{T}{T-1}\right) \qquad (4)$$

Gordon et. al.[2] stated that EVI is described by two parameters, a scale parameter and a location parameter, where the latter is the mode of the distribution. The Extreme Value Type I (EVI) probability distribution function could also be written in the form below:

$$F(x) = \exp\left[-\exp\left(-\left(-\frac{X-u}{\alpha}\right)\right) - x \le x \ge x\right] \qquad (5)$$

The parameters are estimated as:

$$\alpha=\frac{\sqrt{6}s}{\pi} \qquad (6)$$

$$u = X - 0.5772\alpha \qquad (7)$$

A reduced variate y can be defined as:

$$y=\frac{X-u}{\alpha} \qquad (8)$$

According to Hosking and Wallis [7], u is the location parameter and α is the scale parameters. Ponce [5] stated, x is the value of flood discharge and s is the standard deviation. Chow et. al. [10] revealed, substituting the reduced variate into Equation (5) yields:

F(x) = exp [-exp (-y)] (9)

Note that Equation (9) is the same as Equation (1). Solving for y:

$$y=\ln-\left[\ln\left(\frac{1}{F(X)}\right)\right] \qquad (10)$$

Further, according to Chow et.al. [10], values of return period T as an alternate axis to y:

$$\frac{1}{T}=1-P(X-X_T) = 1 - F(xT)$$

So,

$$F(X_T) = \frac{T-1}{T}$$

Chow et. al. [10] then further elaborated that for the EVI distribution, X_T is related to Y_T by Equation (8), or

$$X_T = u + \alpha Y_T \qquad (11)$$

According to Ponce [5] , in the Gumbel Method, values of flood discharge are obtained from the frequency formula:

$$X=\overline{X}+Ks \qquad (12)$$

The frequency factor K is evaluated with the frequency formula:

$$Y=\overline{Y}_n+K\sigma_n \qquad (13)$$

in which y = Gumbel (reduced) variate, a function of return period; yn= the mean of the Gumbel variate; σn = the mean standard deviation of the Gumbel variate; ynand σn and values are a function of record length n. From Equations (12) and Equation (13),

$$X=\overline{X}+\frac{Y-Y_n}{\sigma_n}\sigma \qquad (14)$$

IV. Using the Template

The annual extreme series are arranged in descending order of magnitude. Then the arithmetic mean of the annual flood series is calculated. After that, the plotting position of each sample is determined. In this study, three plotting position formulas are applied onto the samples. The three plotting position formulas are Weibull formula, Gringorten formula and L-moments method [see Equation (8), (9) and (10)]. As to construct the Gumbel distribution by L- moments method with QT/MAF as the y-axis and Gumbel reduce varite (y) as the x-axis, a calculation of L-moments parameters is needed. (Refer to Hosking and Wallis for the details). The parameters are then used as the inputs for the calculations of Gumbel reduced variate, y. The values of annual peak discharge over the arithmetic mean of the annual flood series, Q/MAF or QT/AMAF are then plotted against the reduce variate, y. Finally, dimensionless flood-frequency curve of each individual station was constructed. Then, comparison of

Gumbel distribution by the three plotting positions will be made. Comparison of the two stations using each method will be discussed in this paper.

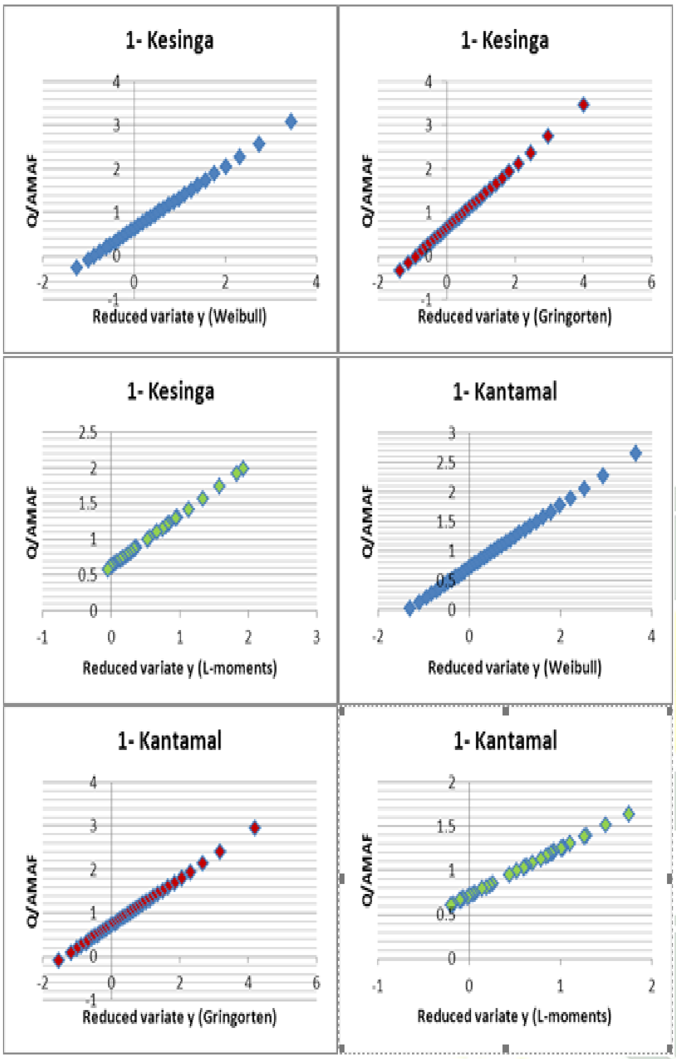
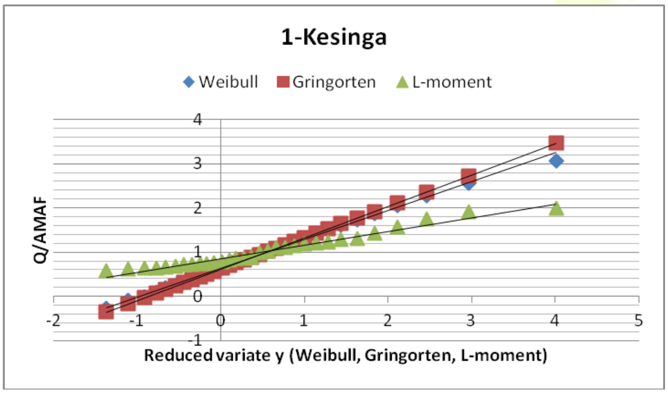


Fig. 3. Gumbel distribution using different plotting position of the Tel sub-basin, India.



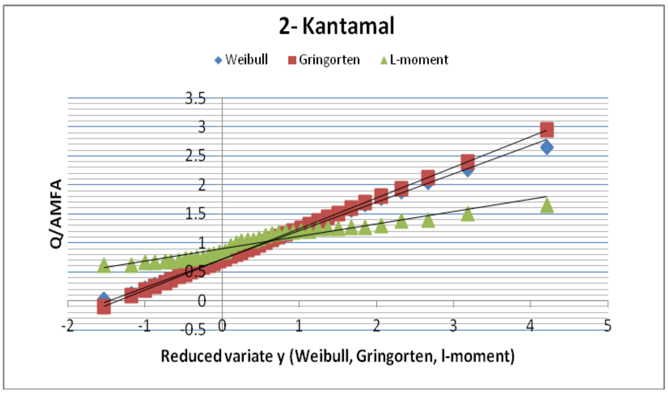


Fig. 4. Gumbel distribution using Weibull, Gringorten and L- moments methods of the Tel sub-basin, India.

The results are utilised to produce the probability plot and flood frequency curves for both the Stations. Figure 3 illustrates the probability plot and flood frequency curve of Gumbel distribution using Weibull, Gringorten and L-moments formulae for both the stations. The discharge and reduce variate (y) data, when plotted together in one graph could contribute on the comparison of the three plotting position methods as shown in figure 4. From the results and analysis of Gumbel distribution using the three plotting position methods, these few trends had been identified: (i) Gumbel distribution with Weibull Formula is always the steepest followed by Gumbel distribution with Gringorten formula and then Gumbel distribution by L-moments method, and (ii) Flood Frequency Curve of Gumbel distribution by L-moments always fits nicely to probability plot compared to the other two cases.

A. Steepness of the Flood Frequency Curve

According to Arnell the steeper the slope of the flow duration curve the greater the variability in flow. Referring to Figure 4, the following equations had been produced in the plot of Gumbel distribution with Weibull, Gringorten and L-moments Formula:

	Kesinga	Kantamal
Weibull:	$y = 0.710x + 0.619$	$y = 0.530x + 0.712$
Gringorten:	$y = 0.651x + 0.632$	$y = 0.489x + 0.724$
L -moments:	$y = 0.308x + 0.854$	$y = 0.213x + 0.898$

It shows that Gumbel distribution with Weibull Formula is always the steepest followed by Gumbel distribution with Gringorten formula and then Gumbel distribution by L-moments method. If we relate these results with the findings from Arnell findings, we could presume that the flow variability for both the stations using Gumbel distribution by Weibull and Gringorten formula is greater than the flow variability of these stations using Gumbel distribution by L-moments formula. According to Hosking, L-moments are less sensitive to variability.

CONCLUSION

In this study, the magnitude and frequency of floods for Tel sub-basin is analysed using Gumbel distribution with three plotting position formulas, namely Weibull, Gringorten and L-moments. Amongst the three methods, L-moments always gives the least ratio of peak discharge of T year's recurrence interval/mean annual flood (QT/MAF) but the appropriateness of L-moments with Gumbel distribution had some limitations. If compared between Weibull and Gringorten formula, Gumbel distribution by Gringorten formula is better than Gumbel distribution by Weibull formula because the former always gives the least ratio. Therefore, it could be concluded that for both stations, L-moments method is the best, but since L-moments method had some limitations, Gringorten formula is still the best plotting position method to be used with Gumbel distribution.

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