Multiplication product rule of THAKKURA PHERU and its generalized rule

[1]S. N. Adhikary, [2]Jeetendra Kumar, [3]Mahadeb Banerjee

[1]Head, [2]Research Scholar, [3]Student
University Department of Mathematics
S.K.M. University
Dumka, Jharkhand, India.

Abstract- **The paper is addressed to the work of the mathematician Thakkura Pheru for obtain a product multiplication consisting of the same digit repeated nine times , where the digit is positive and less than 10. In the present study we generalized the rule which was given by the Indian mathematician Thakkura Pheru.**

Key Words: **Vedic Mathematics, Thakkura pheru, Ganitasarakaumudi (GSK), Multiplication.**

MATHEMATICS SUBJECT CLASSIFICATION: 01A32, 01A05, 01A40

INTRODUCTION: Thakkura Pheru is a great north Indian Mathematician. He was an author of books on Mathematics, coins and gems. Thakkura Pheru gave a rule from his book Ganitasarakaumudi (GSK-4.61) for obtaining a product of multiplication consisting of the same digit repeated nine times.

Viz;

$12345679 \times a \times 9 = aaaaaaaaa$

Where, 'a' is a positive integer and less than 10.

Example – 1. $12345679 \times 7 \times 9 = ?$

Answer: 777777777 ($\therefore$ Nine times 7 by Thakkura Pheru rule)

Example – 2. $12345679 \times 8 \times 9 = ?$

Answer: 888888888 ($\therefore$ Nine times 8 by Thakkura Pheru rule)

Here, we shall try to make a generalized rule for multiplication consisting of the same digit repeated nine times by giving suitable cases.

Case – I: When a = 0 and a $\in$ negative integer

Then, we get

$12345679 \times 0 \times 9 = 000000000$ ($\therefore$ Nine times 0)

Now if, $a \in$ negative integer

Then, we get

$12345679 \times (-a) \times 9 = -\{\, aaaaaaaaa \,\}$

Example – 3. Let a = -6

Then, we get

$12345679 \times (-6) \times 9 = -\,666666666$ ($\therefore$ 9 times 6 with negative sign)

Now the generalized rule –

$$12345679 \times a \times 9 = \begin{cases} aaaaaaaaa, & if \quad 0 \le a \langle 10, \quad a \in positive \quad integer \\ -\{aaaaaaaaa\} & \\ \quad If & -10 \langle a \le 0, \quad a \in negative \quad integer \end{cases}$$

Case – II: When a $\in$ integer which is greater and equal to 10.

i.e., $a \ge 10, \quad a \in positive \quad integer$

$$\begin{Bmatrix} a = 10, & 12345679 \times 10 \times 9 = 1111111110 \\ a = 11, & 12345679 \times 11 \times 9 = 1222222221 \\ a = 12, & 12345679 \times 12 \times 9 = 1333333332 \\ a = 13, & 12345679 \times 13 \times 9 = 1444444443 \\ & \vdots \\ & \vdots \\ a = 18, & 12345679 \times 18 \times 9 = 1999999999 \end{Bmatrix}$$

If we take a = 19, than above rule doesn't work because the sum of digits "a" i.e., (19) must be less than 10.
Here,

1 + 9 = 10 ; this is a contradiction.

Now , we see from 20 to 27, we get

$$\begin{Bmatrix} a = 20, & 12345679 \times 20 \times 9 = 2222222220 \\ a = 21, & 12345679 \times 21 \times 9 = 2333333331 \\ a = 22, & 12345679 \times 22 \times 9 = 2444444442 \\ a = 23, & 12345679 \times 23 \times 9 = 2555555553 \\ & \textit{and so on} \\ a = 27, & 12345679 \times 27 \times 9 = 2999999997 \end{Bmatrix}$$

If we take a = 28 and a = 29 than above rule doesn't work because the sum of digits "a" (i.e., 28 and 29) must be less than 10.
Here,

2 + 8 = 10 and 2 + 9 = 11 ; this is a contradiction.

And so on

Now we establish generalized rule as below-

If a = a_1 , a_2 , where a_1 = 1st term of 'a' and a_2 = 2nd term of 'a'.

Than

$$12345679 \times a \times 9 = \begin{Bmatrix} a_1(a_1 + a_2)............(8\ times)a_2 \\ if\ (a_1 + a_2) \le 9 \end{Bmatrix}$$

i.e.,

$$12345679 \times a \times 9 = \begin{Bmatrix} a_1(a_1 + a_2)............(8\ times)a_2 & 10 \le a \le 18 \\ & 20 \le a \le 27 \\ & 30 \le a \le 36 \\ & \\ & and\quad so\ on \\ & a = 90 \end{Bmatrix}$$

Case – III: When a $\in$ integer which is less and equal to 10.

i.e., $a \le -10, \quad a \in nagetive \quad integer$

Example – 4. $12345679 \times (-21) \times 9 = -2333333331$

Example – 5. $12345679 \times (-34) \times 9 = -3777777774$

If a = - (a_1 , a_2), where a_1 = 1st term of 'a' and a_2 = 2nd term of 'a', (a_1 , $a_2 \in Z^+$).

Than

we establish generalized rule as bellow-

$$12345679 \times a \times 9 = \begin{Bmatrix} -a_1(a_1 + a_2)............(8\ times)a_2 \\ if\ (a_1 + a_2) \le 9 \end{Bmatrix}$$

i.e.,

$$12345679 \times a \times 9 = \left\{ \begin{array}{lr} -\{\ a_1(a_1+a_2)..............(8\ \ times)a_2\}, & a \in [-10,\ -18] \\ & a \in [-20,\ -27] \\ & a \in [-30,\ -36] \\ & \\ & and \quad so\ on \\ & a = -90 \end{array} \right\}$$

Case – IV: **When** $a \in positive \ \ \text{int}\ eger$ **; here a =a₁, a₂ and** $(a_1 + a_2) \geq 10$

$$(\ 12345679 \times 19 \times 9 = 2111111109)$$

$$\begin{pmatrix} 12345679 \times 28 \times 9 = 3111111108 \\ 12345679 \times 29 \times 9 = 3222222219 \end{pmatrix}$$

$$\begin{pmatrix} 12345679 \times 37 \times 9 = 4111111107 \\ 12345679 \times 38 \times 9 = 4222222218 \\ 12345679 \times 39 \times 9 = 4333333329 \end{pmatrix}$$

$$\begin{pmatrix} 12345679 \times 46 \times 9 = 5111111106 \\ 12345679 \times 47 \times 9 = 5222222217 \\ 12345679 \times 48 \times 9 = 5333333328 \\ 12345679 \times 49 \times 9 = 5444444439 \end{pmatrix}$$

$$\begin{pmatrix} 12345679 \times 55 \times 9 = 6111111105 \\ 12345679 \times 56 \times 9 = 6222222216 \\ 12345679 \times 57 \times 9 = 6333333327 \\ 12345679 \times 58 \times 9 = 6444444438 \\ 12345679 \times 59 \times 9 = 6555555549 \end{pmatrix}$$

And so on

Then, we establish generalized rule as bellow-

$$12345679 \times a \times 9 = \begin{pmatrix} (a_1+1)(m_1+m_2).........,\ \ 7\ \ times\ m_2\ \ a_2 \\ where,\ \ m_1 = 1^{st}\ \ term\ of\ \ (a_1+a_2) \\ m_2 = last\ term\ of\ \ (a_1+a_2) \end{pmatrix}$$

Example – 6. $12345679 \times 92 \times 9 = 10222222212$

Example – 7. $12345679 \times 99 \times 9 = 10999999989$

Case – V: **When a** $\in$ **negative integer**

$$such \quad that \quad \begin{array}{l} (a_1+a_2) \geq 10\ \ ; \quad where,\ \ a = -(a_1.a_2) \\ \qquad and\ \ a_1, a_2 \in +ve\ Z \end{array}$$

Example – 8. $12345679 \times -(19) \times 9 = -2111111109$

And so on

Then, we establish generalized rule as bellow-

$$12345679 \times a \times 9 = \left(\begin{array}{c} -\{(a_1+1)(m_1+m_2)........., \ 7 \ times \ m_2 \ a_2\} \\ where, \ m_1 = 1^{st} \ term \ of \ (a_1+a_2) \\ m_2 = last \ term \ of \ (a_1+a_2) \end{array} \right)$$

Example – 9. $12345679 \times -(55) \times 9 = ?$

Answer: here, a = -55

$a_1 = 5$, $a_2 = 5$

$(a_1 + a_2) \geq 10$

$m_1 = 1$, $m_2 = 0$

Required answer is

$$= -\left\{ \ (5=1)(1+0)(1+0)(1+0)(1+0)(1+0)(1+0)(1+0) \ \ 05 \ \right\}$$

$= - \ (\ 6111111105)$.

CONCLUSION

We have framed the generalized rule for different intervals. Hence the multiplication becomes easier when we follow the above rule.

REFERENCES:

[1] Thakura Pheru Ganitasarakaumudi: The moonlight of the Essence of mathematics, editaed with introduction, Translation and Mathematical commentary by Sakhya, Manohar, New Delhi, 2009, pp. xxi-xxii.

[2] D.B.Ojha, "A new property of multiplication magic square" Journal of Emerging Technologies and Innovative Research (JETIR), vol. – 6, No – 2, PP. 892-898, February 2009.

[3] B.S.Yadav and M.Mohan (eds.), Ancient Indian Leaps into Mathematics, published by Birkhasur, New York-2011.

[4] Sreeramula Rajeswar Sarma "Mathematical Literature in the Regionel Languages of India", Indian Journal of History of Science, 28, 40227 (2009), PP. 201-212.

[5] Sreeramula Rajeswara Sarma, "Some Medieval Airthmetical Tables", Indian Journal of History of Science, 32, (1997), 191-198.

[6] Sreeramula Rajeswara Sarma, "Mathematical Literature in Telgu ; An Overview", Sri Venkateswara University Oriental Journal, 28, 7790, (1985).